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TAYLOR'S HYDRODYNAMICS OF STRONG SHOCKS APPLIED TO GASES HAVING SMALL VALUES OF $\gamma-1$

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ABSTRACT

When $\gamma - 1$ is small, the pressure and density gradients become very large at the shock front and very small a short distance behind the shock front. Bethe has made use of this behavior in developing a set of analytical solutions to the hydrodynamical equations which become more nearly valid as γ approaches one. In the second section, direct point-by-point numerical calculations of Taylor's equations are carried out for $\gamma = 1.2$ and 1.1. From these it may be seen that Bethe's approximate solutions give very good results for values of γ even larger than 1.2.

[redacted]

TAYLOR'S HYDRODYNAMICS OF STRONG SHOCKS APPLIED TO GASES HAVING SMALL VALUES OF γ-1

G. I. Taylor has developed a theory for the hydrodynamics of strong shock waves arising from a point explosion (BM-35; RC-210). This treatment assumes similarity, i.e. the pressure, density, and material velocity behind the shock are functions of the radius of the shock wave and the ratio of the radius of the point under consideration to the radius of the shock wave. When γ-1 is small, the pressure drops from its value at the shock front to approximately half this value and the density goes from its value at the shock front to almost zero in a very short distance behind the shock front. Thus the problem may be divided into a consideration of the thin shell in the neighborhood of the shock front and a consideration of the central core in which the conditions are relatively simple. We obtain approximate solutions to the Taylor equations which become good for small values of γ-1. The Taylor conditions in the neighborhood of the shock front may be used even when the conditions in the central core are quite different from those arising from the point explosion, i.e., the simple similarity conditions which Taylor assumes are no longer valid.

I. LIMITING FORM DEVELOPED BY BETHE OF TAYLOR'S HYDRODYNAMICS OF STRONG SHOCKS IN GASES WITH SMALL VALUES OF γ-1

The following equations were obtained by Taylor for strong shock waves in gases having any value of γ-1:

$$\eta = r/R \qquad (1)$$

$$p/p_o = y = \frac{A^2 f(\eta)}{c_o^2 R^3} \qquad (2)$$

$$\rho/\rho_o = \psi(\eta) \qquad (3)$$

[redacted]

[redacted]

$$u = AR^{-3/2}\,\phi(\eta) \qquad (4)$$

$$U = AR^{-3/2} \qquad (5)$$

$$E_{tot} = 4\pi\rho_0 A^2 \int_0^1 \left[(1/2)\psi\phi^2 + \frac{f}{\gamma(\gamma-1)}\right]\eta^2 d\eta \qquad (6)$$

Here r is the radius of a point in question; R is the radius of the shock front; the subscript zero refers to the undisturbed atmosphere or conditions in front of the shock wave; p is the pressure; ρ is the density; c is the velocity of sound, $c^2 = \gamma p/\rho$; u is the material velocity; U is the shock wave velocity or dR/dt; E_{tot} is the total energy of the explosion; A is a constant to be determined in terms of the total energy of the explosion, the velocity of the shock wave, or the shock pressure when the shock reaches a particular value of the radius; f, ϕ, and ψ are functions to be determined by the following differential equations:

$$\frac{f'}{f} = \frac{-3(\eta-\phi) + (\gamma/2)\,\phi - 2\gamma\phi^2/\eta}{(\eta-\phi)^2 - f/\psi} \qquad (7)$$

$$\phi' = \frac{f'/(\gamma\psi) - 3\phi/2}{\eta-\phi} \qquad (8)$$

$$\frac{\psi'}{\psi} = \frac{\phi' + 2\phi/\eta}{\eta-\phi} \qquad (9)$$

[redacted]

Or eliminating f' and ϕ' from Eqs. (8) and (9):

$$\phi' = \left[-2\,\phi/\eta + 3/\gamma + (3/2)\,\phi\,(\psi/f)\,(\eta-\phi)\right] \Big/ \left[1-(\psi/f)\,(\eta-\phi)^2\right] \tag{8a}$$

$$\psi'/\psi = \left[\frac{3}{\gamma(\eta-\phi)} + (3/2)\,\phi\,\psi/f - \frac{2\phi\psi}{\eta f}\,(\eta-\phi)\right] \Big/ \left[1-(\psi/f)\,(\eta-\phi)^2\right] \tag{9a}$$

At the <u>shock front, $\eta = 1$</u> , f, ϕ , and ψ have the values:

$$f(1) = 2\gamma/(\gamma+1) \tag{10}$$

$$\phi(1) = 2/(\gamma + 1) \tag{11}$$

$$\psi(1) = (\gamma + 1)/(\gamma-1) \tag{12}$$

And substituting (10), (11), and (12) into Eqs. (7), (8), and (9) we obtain for the values of the derivatives f' , ϕ' , and ψ' at the shock front:

$$f'(1) = \frac{2\gamma(2\gamma^2 + 7\gamma - 3)}{(\gamma-1)\,(\gamma + 1)^2} \tag{13}$$

$$\phi'(1) = \frac{\gamma + 9}{(\gamma + 1)^2} \tag{14}$$

$$\psi'(1) = \frac{5\gamma + 13}{(\gamma-1)^2} \tag{15}$$

<u>In the core</u>, i.e. for sufficiently small values of η (for gamma less than 1.2, η = .9 is already sufficiently small):

$$f \doteq \text{constant} \tag{16}$$

$$\phi \doteq \eta/\gamma \tag{17}$$

$$\psi \doteq 0 \tag{18}$$

[redacted]

[redacted]

Knowing the behavior of these functions both in the region of the shock front and in the core, we can find approximations for the right hand side of Eqs. (7), (8), and (9) which are valid for small values of $\gamma - 1$ and sufficiently simple so that these equations may be integrated for values of γ near unity. First, we must establish the following propositions:

1). $\eta - \phi$ is always proportional to $(\gamma-1)$

At the shock front, $\eta - \phi = (\gamma-1)/(\gamma+1)$ (19)

In the core, $\eta - \phi = \eta(\gamma-1)/\gamma$ (20)

Therefore it may be assumed that the proportionality applies for all values of η.

2). $f' = 3\eta\psi/2$ (21)

At the shock front where $\eta = 1$: $f' = \dfrac{2\gamma(2\gamma^2 + 7\gamma - 3)}{(\gamma + 1)^3}\,\psi$ (22)

If $\gamma = 1$, Eq. (21) is exactly satisfied at the shock front; if $\gamma = 1.2$, at the shock front, $f' = 1.866\,\psi$. (23)

In the core, using Eqs. (17) and (20), Eq. (7) becomes:

$$f'/f = + \frac{(3/\gamma)\eta(\gamma-1) - \eta/2 + 2\eta/\gamma}{(f/\psi) - [(\eta/\gamma)(\gamma-1)]^2}$$

$$\doteq (\psi/f)\left[\eta\left(2/\gamma - 1/2 + (3/\gamma)(\gamma-1)\right)\right] \doteq (3/2)\eta\,\psi/f \qquad (24)$$

Again the proposition (21) is exactly satisfied if $\gamma = 1$ and very nearly satisfied if $\gamma = 1.2$. Thus we can assume that the proposition is approximately true for all values of η.

3). $(\psi/f)(\eta-\phi)^2 \ll 1$ (25)

At the shock front:

$(\psi/f)(\eta-\phi)^2 = (\gamma-1)/2\gamma$ (26)

[redacted]

[redacted]

And in the core $(\psi/f)\,(\eta-\phi)^2$ is virtually zero because of Eqs. (16) and (18). Thus we assume that this quantity is always small.

4). $\phi \doteq \eta/\gamma$ (27)

At the shock front, $\phi = 2/(\gamma + 1)$. If γ is exactly one, this agrees with the proposition (27), otherwise it differs by terms of the order of $(\gamma-1)$. For example if $\gamma = 1.2$, the $\phi = .909$ instead of $.833$ as expected from (27). In the core, Taylor has shown that (27) is an excellent approximation for any value of γ.

At the shock front, $\psi = (\gamma + 1)/(\gamma - 1)$ and is very large for small values of γ. It rapidly becomes smaller with decreasing η and approaches zero in the core. In practically all of the interesting region of the shock shell, ψ is greater than unity. In this region, η is practically unity. By the proposition (25), the denominators of the right hand sides of Eqs. (8a) and of (9a) are both unity, also the third term in the numerator of Eq. (9a) is small compared to the first two. Thus the Eqs. (8a) and (9a) become:

$$\phi' \doteq -2\,\phi/\eta + 3/\gamma + (3/2)\,\phi\,(\psi/f)\,(\eta-\phi)$$

$$\doteq 1/\gamma + \frac{3\eta}{2\gamma}\,\frac{\psi}{f}\,(\eta-\phi)$$

$$\doteq 1 + (3/2)\,(\psi/f)\,(\eta-\phi) \qquad (8b)$$

$$\frac{\psi'}{\psi} \doteq \frac{3}{\gamma(\eta-\phi)} + \left(\frac{3}{2}\right)\left(\frac{\eta}{\gamma}\right)\left(\frac{\psi}{f}\right)$$

$$\doteq 3/(\eta-\phi) + (3/2)\,\psi/f \qquad (9b)$$

The justification for the last approximation in obtaining Eq. (8b) depends on ψ being larger than unity. The third equation necessary to solve for f, ϕ, and ψ in the shock shell is Eq. (21):

$$f' \doteq (3/2)\,\psi \qquad (21b)$$

[redacted]

It is convenient to let:

$$\eta - 1 = (\gamma-1)x \tag{28}$$

$$\eta - \phi = (\gamma-1)w \tag{29}$$

Then Eqs. (8b), (9b), and (21b) become:

$$dw/dx = 1 - \phi' = - (3/2)(\gamma-1)(\psi/f)\,w \tag{8c}$$

$$d \log \psi/dx = 3/w + (3/2)(\psi/f)(\gamma-1) \tag{9c}$$

$$df/dx = (3/2)(\gamma-1)\,\psi \tag{21c}$$

Combining Eqs. (8c) and 21c):

$$dw/dx = - (w/f)\,df/dx \tag{29}$$

or

$$\frac{wdf}{fdw} = -1 = \frac{d \log f}{d \log w} \tag{30}$$

And integrating:

$$\log f = - \log w + \text{constant} \tag{31}$$

or

$$fw = \text{constant} = f(\eta=1)\,w(\eta=1) = \left(\frac{2\gamma}{\gamma+1}\right)\frac{1}{\gamma+1} \approx 1/2 \tag{32}$$

Then combining Eqs (9c) and (21c) and making use of Eq. (32):

$$d \log \psi/dx = 6f + d \log f/dx \tag{33}$$

Now let:

$$Z = \left[(\gamma-1)/2\right]\,\psi/f \tag{34}$$

Then:

$$d \log Z/dx = 6f \tag{35}$$

And Eq. (21c) becomes:

$$d \log f/dx = 3Z \tag{36}$$

[redacted]

Combining Eqs. (35) and (36):

$$\frac{d \log Z}{d \log f} = \frac{f dZ}{Z df} = 2 f/Z \tag{37}$$

or

$$dZ/df = 2 \tag{38}$$

So that on integrating:

$$f = (1/2)Z + \text{constant} \tag{39}$$

Since $f(\eta=1) = 2\gamma/(\gamma+1) = 1$ and $Z(\eta=1) = \left[(\gamma-1)/2\right]\left[2/(\gamma-1)\right] f(\eta=1) = 1$

Thus:

$$f = 1/2(Z+1) \tag{40}$$

Substituting this back into Eq. (35):

$$dZ/dx = 3Z(Z+1) \tag{41}$$

And integrating:

$$3x = \int \frac{dZ}{Z(Z+1)} = \log\ Z/(Z+1) + \text{constant} \tag{42}$$

And at the shock front $x = 0$, $Z = 1$ so that:

$$3x = \log_e \frac{2Z}{Z+1} \tag{43}$$

or

$$Z = e^{3x}/(2-e^{3x}) \tag{44}$$

These equations then determine f, ϕ, and ψ. The complete solution to Taylor's equations for small values of $\gamma-1$ in the region of the shock shell can be obtained from the following expressions for f, ϕ, and ψ:

$$f = 1/\left\{2-e^{3(\eta-1)/(\gamma-1)}\right\} \tag{45}$$

$$\phi = \eta-(\gamma-1)\left[1-(1/2)e^{3(\eta-1)/(\gamma-1)}\right] \tag{46}$$

$$\psi = \frac{2e^{3(\eta-1)/(\gamma-1)}}{\gamma-1} \Big/ \left[2 - e^{3(\eta-1)/(\gamma-1)}\right]^2 \tag{47}$$

[redacted]

 [redacted]

From Eq. (45) it appears that f approaches the value 0.50 in the core. This result might be questioned since the approximations used are no longer valid in the core. However, direct numerical integrations for gamma equal to 1.1 and 1.2 indicate that in the core $f \doteq 0.50 - (\gamma-1)^2/\gamma$.

II. NUMERICAL SOLUTION OF TAYLOR'S EQUATIONS WITH $\gamma = 1.4^*$, 1.2 and 1.1

We made a straight point-by-point numerical integration of Taylor's equations for $\gamma = 1.2$ and 1.1 . The values of f, ϕ and ψ so obtained are given in tables I and II, and shown in Figs. 1 and 2. The following values were obtained for the solutions of the equations.

	$\gamma = 1.4^*$	$\gamma = 1.2$	$\gamma = 1.1$
$\int_0^1 \psi\phi^2\eta^2 d\eta$	0.185	0.2374	0.2673
$\int_0^1 f\eta^2 d\eta$	0.187	0.1766	0.1736
$\int_0^1 f^{1/\gamma}\eta^2 d\eta$	0.219	0.1941	0.1847
$\int_0^1 \psi\eta^2 d\eta$	0.3333	0.3275	0.3167

* All values for $\gamma = 1.4$ were taken from (BM-35; RC-210) by G.I. Taylor.

[redacted]

γ	E ergs	ε tons TNT	A $cm^{5/2}$ sec^{-1}
1.4*	$5.36\ \rho_o A^2$	$1.2808 \times 10^{-16} \rho_o A^2$	2.4602×10[illegible]$\varepsilon^{1/2}$
1.2	10.742	2.5668	1.7378
1.1	21.520	5.1422	1.2279

γ	p/p_o	t sec	E_{kin}/E_{int}
1.4*	$5.677 \times 10^9 R^{-3} f(\eta) \varepsilon$	$.1626 \times 10^{-9} R^{5/2} \varepsilon^{-\frac{1}{2}}$	.2774
1.2	3.2466	.2302	.1613
1.1	1.768	.3258	.0847

(A convenient unit for energy is the "ton of TNT" equal to 4.185×10^{16} ergs.)

Here p_o has been taken as one atmosphere and $\rho_o = 1.29 \times 10^{-3} gms/cm^3$.

In calculating the energy loss we should point out an error in Taylor's paper (BM-35; RC-210) which was first noticed by William Penney. The energy in the system after the blast wave has passed should be estimated from the enthalpy instead of the internal energy and therefore Taylor's energy losses, E_1 , should be multiplied by gamma. With this in mind, we obtain for the fraction of energy lost up to the time that the blast pressure is $y_1 = p/p_o$:

$\gamma = 1.4$

$$E_1 \text{ (corrected)}/E = (1/y_1)\ (1.341\ \ y_1^{(1/1.4)}\ - 2.282)$$

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*All values for $\gamma = 1.4$ were taken from (BM-35; RC-210) by G.I. Taylor.

$\gamma = 1.2$

$$E_1 \text{ (corrected)}/E = (1/y_1)\ (1.152\ \ y_1^{(1/1.2)}\ - 2.090)$$

$\gamma = 1.1$

$$E_1 \text{ (corrected)}/E = (1/y_1)\ (1.084\ \ y_1^{(1/1.1)}\ - 1.937)$$

The second term in these equations is due to the intrinsic energy of the initial undisturbed gas. If we intend to use these relations for hot air, then it is clear that the intrinsic energy of the initial cold undisturbed air corresponds to $\gamma = 1.4$. In this case:*

$\gamma = 1.2$ (hot air)

$$\frac{E_1\text{(corrected)}}{E} = (1/y_1)\ \left[1.152\ y_1^{1/1.2}\ - 1.241\right]$$

$\gamma = 1.1$ (hot air)

$$\frac{E_1\text{(corrected)}}{E} = (1/y_1)\ \left[1.084\ y_1^{.909} - 0.649\right]$$

The results are shown in Table III. The energy loss becomes larger as gamma becomes smaller. Below 20 atmospheres shock pressure, the shock can no longer be considered as strong and the Taylor solutions no longer are valid.

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* If we let $E_1/E = (1/y_1)\left[a y_1^{1/\gamma} - \beta\right]$, then β/y_1 is just the heat content of the air within the sphere of radius R divided by E. Or

$$\frac{\beta}{y_1} = \frac{(1.4)4\pi p_o R^3}{3(.4)Ey_1}$$

But $$R^3/y_1 = A^2 f(\eta = 1)/c_o^2$$

So that

$$\beta/y_1 = 29.322/\left[(\gamma + 1)\ (E/A^2\rho_o)\right]$$

[redacted]

[redacted]

TABLE I

$\gamma = 1.2$

η	ξ	ϕ	ψ	η	ξ	ϕ	ψ
1.0000	1.0909	0.9091	11.0000	.9705	0.7322	0.8541	[illegible]
.9995	1.0806	0.9080	10.7625	.9700	0.7294	0.8533	3.7083
.9990	1.0706	0.9070	10.5322	.9695	0.7260	0.8525	3.7414
.9985	1.0608	0.9060	10.3087	.9690	0.7227	0.8517	[illegible]
.9980	1.0514	0.9050	10.0918	.9685	0.7195	0.8509	3.6342
.9975	1.0418	0.9040	9.8813	.9680	0.7163	0.8501	3.5921
.9970	1.0326	0.9030	9.6767	.9675	0.7132	0.8493	3.5310
.9965	1.0236	0.9020	9.4777	.9670	0.7101	0.8485	[illegible]
.9960	1.0148	0.9010	9.2842	.9665	0.7070	0.8477	[illegible]
.9955	1.0062	0.9000	9.0961	.9660	0.7040	[illegible]	[illegible]
.9950	0.9978	0.8990	8.9134	.9655	0.7010	[illegible]	[illegible]
.9945	0.9896	0.8980	8.7360	.9650	0.6981	[illegible]	[illegible]
.9940	0.9816	0.8970	8.5639	.9645	0.6952	[illegible]	[illegible]
.9935	0.9737	0.8960	8.3962	.9640	0.6924	0.8437	3.1933
.9930	0.9659	0.8950	8.2329	.9635	0.6896	0.8429	3.1548
.9925	0.9583	0.8940	8.0738	.9630	0.6869	0.8421	3.1116
.9920	0.9509	0.8930	7.9191	.9625	0.6842	0.8413	3.0692
.9915	0.9435	0.8920	7.7685	.9620	0.6815	0.8405	3.0276
.9910	0.9364	0.8910	7.6219	.9615	0.6789	0.8397	2.9867
.9905	0.9294	0.8900	7.4791	.9610	0.6763	0.8389	2.9466
.9900	0.9225	0.8890	7.3400	.9605	0.6737	0.8381	2.9072
.9895	0.9158	0.8880	7.2045				
.9890	0.9092	0.8870	7.0725	.960	0.6712	0.8373	2.8684
.9885	0.9027	0.8860	6.9438	.959	0.6662	0.8358	2.7926
.9880	0.8963	0.8850	6.8184	.958	0.6614	0.8343	2.7192
.9875	0.8901	0.8840	6.6961	.957	0.6567	0.8328	2.6482
.9870	0.8840	0.8831	6.5769	.956	0.6521	0.8313	2.5795
.9865	0.8780	0.8822	6.4606	.955	0.6477	0.8298	2.5131
.9860	0.8721	0.8813	6.3471	.954	0.6434	0.8283	2.4489
.9855	0.8663	0.8804	6.2363	.953	0.6392	0.8269	2.3869
.9850	0.8606	0.8795	6.1281	.952	0.6351	0.8255	2.3270
.9845	0.8550	0.8786	6.0224	.951	0.6311	0.8241	2.2690
.9840	0.8495	0.8777	5.9192	.950	0.6272	0.8227	2.2128
.9835	0.8441	0.8768	5.8184	.949	0.6234	0.8213	2.1583
.9830	0.8388	0.8759	5.7200	.948	0.6197	0.8199	2.1055
.9825	0.8336	0.8750	5.6238	.947	0.6161	0.8185	2.0543
.9820	0.8285	0.8741	5.5298	.946	0.6126	0.8171	2.0047
.9815	0.8235	0.8732	5.4379	.945	0.6092	0.8157	1.9566
.9810	0.8186	0.8723	5.3481	.944	0.6059	0.8143	1.9100
.9805	0.8138	0.8714	5.2603	.943	0.6027	0.8130	1.8648
.9800	0.8091	0.8705	5.1745	.942	0.5996	0.8117	1.8210
.9795	0.8045	0.8696	5.0906	.941	0.5966	0.8104	1.7785
.9790	0.7999	0.8687	5.0086	.940	0.5936	0.8091	1.7372
.9785	0.7954	0.8678	4.9284	.939	0.5907	0.8078	1.6970
.9780	0.7910	0.8669	4.8499	.938	0.5879	0.8065	1.6579
.9775	0.7867	0.8660	4.7731	.937	0.5852	0.8052	1.6199
.9770	0.7824	0.8651	4.6980	.936	0.5825	0.8039	1.5830
.9765	0.7782	0.8642	4.6245	.935	0.5799	0.8026	1.5471
.9760	0.7741	0.8633	4.5526	.934	0.5774	0.8013	1.5122
.9755	0.7700	0.8624	4.4822	.933	0.5749	0.8000	1.4783
.9750	0.7660	0.8615	4.4133	.932	0.5725	0.7987	1.4453
.9745	0.7621	0.8606	4.3458	.931	0.5701	0.7975	1.4132
.9740	0.7582	0.8597	4.2797				
.9735	0.7544	0.8589	4.2150	.930	0.5678	0.7963	1.3820
.9730	0.7507	0.8581	4.1516	.928	0.5633	0.7939	1.3219
.9725	0.7470	0.8573	4.0894	.926	0.5590	0.7915	1.2647
.9720	0.7434	0.8565	4.0285	.924	0.5549	0.7891	1.2102
.9715	0.7398	0.8557	3.9688	.922	0.5510	0.7867	1.1582
.9710	0.7363	0.8549	3.9103	.920	0.5473	0.7844	1.1087

[redacted]

TABLE I--CONCLUDED

$\gamma = 1.2$

η	ξ	ϕ	ψ
.918	0.5438	0.7821	1.0617
.916	0.5404	0.7798	1.0171
.914	0.5372	0.7775	0.9747
.912	0.5341	0.7752	0.9343
.910	0.5312	0.7730	0.8958
.908	0.5283	0.7708	0.8591
.906	0.5257	0.7686	0.8241
.904	0.5231	0.7664	0.7907
.902	0.5206	0.7643	0.7589
.900	0.5182	0.7622	0.7286
.896	0.5137	0.7580	0.6705
.892	0.5096	0.7538	0.6174
.888	0.5058	0.7497	0.5688
.884	0.5023	0.7457	0.5244
.880	0.4991	0.7417	0.4837
.876	0.4962	0.7478	0.4464
.872	0.4935	0.7339	0.4121
.868	0.4910	0.7301	0.3806
.864	0.4887	0.7263	0.3515
.860	0.4863	0.7225	0.3248
.856	0.4847	0.7188	0.3001
.852	0.4830	0.7151	0.2773
.848	0.4815	0.7114	0.2563
.844	0.4801	0.7078	0.2369
.840	0.47875	0.70419	0.21900
.832	0.47627	0.69702	0.18580
.824	0.47419	0.68993	0.15759
.816	0.47245	0.68292	0.13360
.808	0.47099	0.67597	0.11318
.800	0.46976	0.66907	0.09579
.792	0.46873	0.66221	0.08098
.784	0.46737	0.65539	0.06837
.776	0.46715	0.64860	0.05764
.768	0.46655	0.64184	0.04852
.76	0.46605	0.63510	0.04073
.74	0.46501	0.61829	0.02439
.72	0.46441	0.60158	0.01434
.70	0.46407	0.58493	0.00828
.68	0.46388	0.56831	0.00468
.66	0.463774	0.551719	0.002584
.64	0.463717	0.535145	0.001391
.62	0.463687	0.518586	0.000728
.60	0.463671	0.502041	0.000369
.58	0.463664	0.485509	0.000181
.56	0.463661	0.468990	0.000085
.54	0.463659	0.452484	0.000038
.52	0.463658	0.435991	0.000016
.5	0.463658	0.419512	0.000006
.4	0.463658	0.337317	0.000000
.3	0.463658	0.256597	0
.2	0.463658	0.167918	0
.1	0.463658	0.089407	0
0	0.463658	0	0

TABLE II

$\gamma = 1.1$

η	ξ	ϕ	ψ
1.0000	1.0476	0.9524	21.0000
.9995	1.0298	0.9513	20.0747
.9990	1.0129	0.9502	19.2062
.9985	0.9966	0.9491	18.3901
.9980	0.9811	0.9480	17.6223
.9975	0.9662	0.9469	16.8993
.9970	0.9520	0.9458	16.2177
.9965	0.9384	0.9447	15.5745
.9960	0.9253	0.9436	14.9670
.9955	0.9127	0.9425	14.3927
.9950	0.9006	0.9414	13.8492
.9945	0.8890	0.9404	13.3346
.9940	0.8773	0.9394	12.8462
.9935	0.8671	0.9384	12.3824
.9930	0.8568	0.9374	11.9416
.9925	0.8468	0.9364	11.5224
.9920	0.8372	0.9354	11.1235
.9915	0.8279	0.9344	10.7435
.9910	0.8190	0.9334	10.3815
.9905	0.8104	0.9324	10.0362
.9900	0.8021	0.9314	9.7068
.989	0.7860	0.9295	9.0777
.988	0.7710	0.9276	8.5037
.987	0.7569	0.9257	7.9787
.986	0.7437	0.9239	7.4975
.985	0.7314	0.9221	7.0551
.984	0.7198	0.9203	6.5475
.982	0.6980	0.9168	5.8951
.980	0.6787	0.9134	5.2540
.978	0.6616	0.9101	4.7033
.976	0.6464	0.9069	4.2271
.974	0.6328	0.9038	3.8125
.972	0.6206	0.9008	3.4492
.970	0.6096	0.8979	3.1292
.968	0.5996	0.8951	2.8460
.966	0.5906	0.8923	2.5941
.964	0.5824	0.8896	2.3694
.962	0.5749	0.8870	2.1681
.960	0.5681	0.8844	1.9871
.958	0.5619	0.8819	1.8239
.954	0.5505	0.8770	1.5289
.950	0.5410	0.8722	1.2894
.946	0.5331	0.8676	1.0906
.942	0.5264	0.8631	0.9264
.938	0.5208	0.8588	0.7893
.934	0.5160	0.8546	0.6740
.930	0.5120	0.8504	0.5766
.926	0.5086	0.8463	0.4940
.922	0.5057	0.8423	0.4238
.918	0.5032	0.8383	0.3639
.914	0.5011	0.8344	0.3127
.910	0.4993	0.8305	0.2688
.906	0.4977	0.8267	0.2311

[redacted]

[redacted]

TABLE II--Concluded

$\gamma = 1.1$

η	f	ϕ	ψ
.898	0.4950	0.8181	0.16[illegible]4
.890	0.4931	0.8113	0.1193
.882	0.4917	0.8042	0.0868
.874	0.4907	0.7968	0.0615
.866	0.4900	0.7894	0.0442
.858	0.4895	0.7820	0.0315
.850	0.4892	0.7748	0.0225
.842	0.4890	0.7673	0.0159
.834	0.4888	0.7601	0.0113
.826	0.4887	0.7529	0.0080
.818	0.4886	0.7459	0.0056
.810	0.4885	0.7385	0.0039
.794	0.4884	0.7240	0.00152
.778	0.4884	0.7095	0.0005714
.762	0.4884	0.6951	0.0002076
.746	0.4884	0.6806	0.0000722
.70	0.4884	0.6364	0
.65	0.4884	0.5909	0
.60	0.4884	0.5465	0
.55	0.4884	0.5000	0
.50	0.4884	0.4545	0
.4	0.4884	0.3636	0
.3	0.4884	0.2727	0
.2	0.4884	0.1818	0
.1	0.4884	0.09091	0
0	0.4884	0	0

TABLE III

Corrected Fraction of Energy Lost when Blast Wave Has Pressure y_1

y_1	E_1(corrected)/E			[E_1(corrected)/E] hot air		
	γ=1.4	γ=1.2	γ=1.1	γ=1.4	γ=1.2	γ=1.1
10,000 atm	.097	.248	.469	.097	.248	.469
1,000	.185	.362	.576	.185	.363	.577
100	.336	.514	.693	.336	.522	.706
50	.393	.558	.721	.393	.575	.747
20	.455	.595	.728	.455	.637	.792

[redacted]

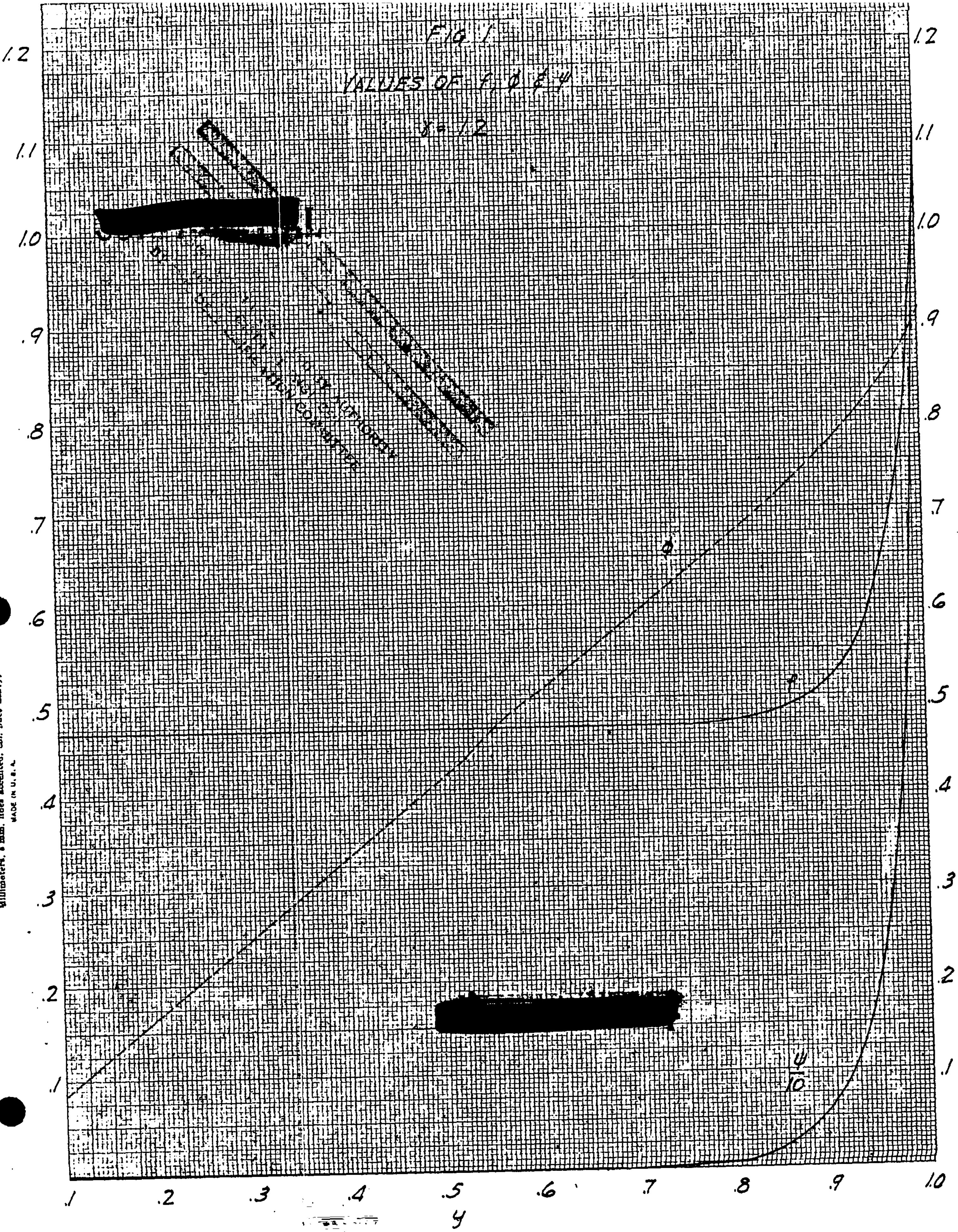
FIG 1
VALUES OF f, φ & ψ
γ = 1.2
φ
f
ψ/10
y
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2

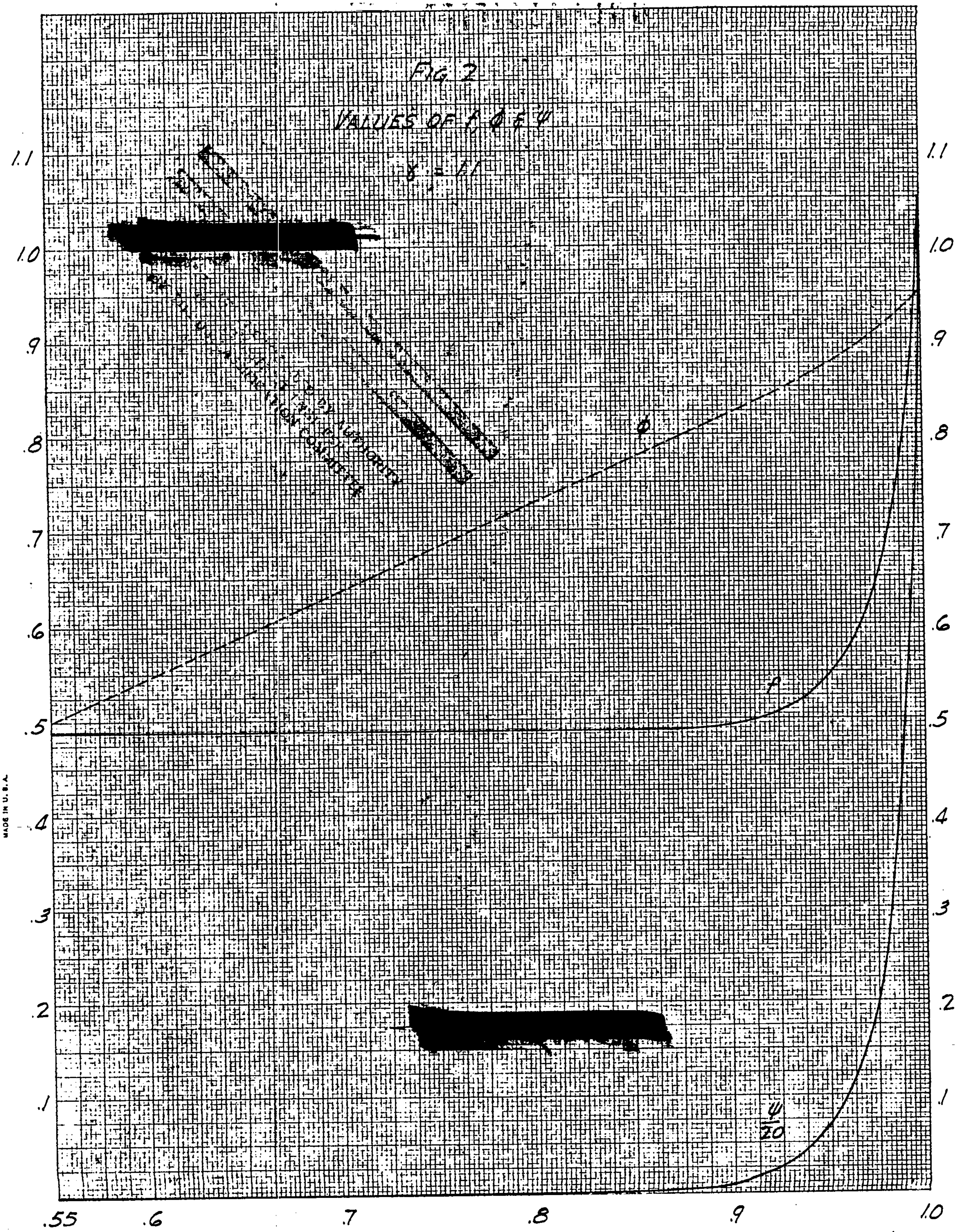

Fig 2
VALUES OF f, Φ & Ψ
γ = 1.1
[redacted]
Φ
f
Ψ/20
[redacted]
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.55
.6
.7
.8
.9
1.0

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